Mesh Decimation

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Applications

• Oversampled 3D scan data

~150k triangles

~80k triangles
Applications

- Overtessellation: E.g. iso-surface extraction
Applications

• Multi-resolution hierarchies for
  – efficient geometry processing
  – level-of-detail (LOD) rendering
Applications

• Adaptation to hardware capabilities
Size-Quality Tradeoff

error

size
Outline

• Applications

• Problem Statement

• Mesh Decimation Methods
  – Vertex Clustering
  – Iterative Decimation
  – Extensions
  – Remeshing
  – Variational Shape Approximation
Problem Statement

• Given: \( M = (V, F) \)

• Find: \( M' = (V', F') \) such that

1. \(|V'| = n < |V|\) and \( \|M - M'\| \) is minimal, or

2. \( \|M - M'\| < \epsilon \) and \(|V'|\) is minimal
Problem Statement

• Given: $\mathcal{M} = (\mathcal{V}, \mathcal{F})$

• Find: $\mathcal{M}' = (\mathcal{V}', \mathcal{F}')$ such that

1. $|\mathcal{V}'| = n < |\mathcal{V}|$ and $\|\mathcal{M} - \mathcal{M}'\|$ is minimal, or

2. $\|\mathcal{M} - \mathcal{M}'\| < \epsilon$ and $|\mathcal{V}'|$ is minimal

hard!

→ look for sub-optimal solution
Problem Statement

• Given: $\mathcal{M} = (V, F)$

• Find: $\mathcal{M}’ = (V’, F’) \text{ such that}$

1. $|V'| = n < |V|$ and $\|\mathcal{M} - \mathcal{M}'\|$ is minimal, or

2. $\|\mathcal{M} - \mathcal{M}'\| < \epsilon$ and $|V'|$ is minimal

• Respect additional fairness criteria
  – normal deviation, triangle shape, scalar attributes, etc.
Outline

• Applications
• Problem Statement
• Mesh Decimation Methods
  – Vertex Clustering
  – Iterative Decimation
  – Extensions
Vertex Clustering

- Cluster Generation
- Computing a representative
- Mesh generation
- Topology changes
Vertex Clustering

- Cluster Generation
  - Uniform 3D grid
  - Map vertices to cluster cells

- Computing a representative

- Mesh generation

- Topology changes
Vertex Clustering

• Cluster Generation
  – Hierarchical approach
  – Top-down or bottom-up

• Computing a representative

• Mesh generation

• Topology changes
Vertex Clustering

• Cluster Generation

• Computing a representative
  – Average/median vertex position
  – Error quadrics

• Mesh generation

• Topology changes
Computing a Representative

Average vertex position $\rightarrow$ Low-pass filter
Computing a Representative Median vertex position $\rightarrow$ Sub-sampling
Computing a Representative

Error quadrics
Error Quadrics

- Squared distance to plane

\[ p = (x, y, z, 1)^T, \quad q = (a, b, c, d)^T \]

\[ \text{dist}(q, p)^2 = (q^T p)^2 = p^T (qq^T) p =: p^T Q_q p \]

\[ Q_q = \begin{bmatrix}
    a^2 & ab & ac & ad \\
    ab & b^2 & bc & bd \\
    ac & bc & b^2 & cd \\
    ad & bd & cd & d^2
\end{bmatrix} \]
Error Quadrics

• Sum distances to vertex’ planes

\[
\sum_i \text{dist}(q_i, p)^2 = \sum_i p^T Q_{q_i} p = p^T \left( \sum_i Q_{q_i} \right) p =: p^T Qp p
\]

• Point that minimizes the error

\[
\begin{bmatrix}
q_{11} & q_{12} & q_{13} & q_{14} \\
q_{21} & q_{22} & q_{23} & q_{24} \\
q_{31} & q_{32} & q_{33} & q_{34} \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
p^* = \begin{bmatrix}
0 \\
0 \\
0 \\
1
\end{bmatrix}
\]
Comparison

average

median

error quadric
Vertex Clustering

- Cluster Generation
- Computing a representative
- Mesh generation
  - Clusters $p \leftrightarrow \{p_0, ..., p_n\}$, $q \leftrightarrow \{q_0, ..., q_m\}$
  - Connect $(p,q)$ if there was an edge $(p_i,q_j)$
- Topology changes
Vertex Clustering

• Cluster Generation
• Computing a representative
• Mesh generation

• Topology changes
  – If different sheets pass through one cell
  – Not manifold
Outline

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• Mesh Decimation Methods
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  – Extensions
Incremental Decimation

• General Setup
  • Decimation operators
  • Error metrics
  • Fairness criteria
  • Topology changes
General Setup

Repeat:

pick mesh region
apply decimation operator

Until no further reduction possible
Greedy Optimization

For each region
   evaluate quality after decimation
   enque(quality, region)

Repeat:
   pick best mesh region
   apply decimation operator
   update queue
Until no further reduction possible
Global Error Control

For each region
   evaluate quality after decimation
   enqueue(quality, region)

Repeat:
   pick best mesh region
   if error < ε
      apply decimation operator
      update queue
Until no further reduction possible
Incremental Decimation

- General Setup
- Decimation operators
- Error metrics
- Fairness criteria
- Topology changes
Decimation Operators

• What is a "region"?
• What are the DOF for re-triangulation?
• Classification
  – Topology-changing vs. topology-preserving
  – Subsampling vs. filtering
  – Inverse operation → progressive meshes
Vertex Removal

Select a vertex to be eliminated
Vertex Removal

Select all triangles sharing this vertex
Vertex Removal

Remove the selected triangles, creating the hole
Vertex Removal

Fill the hole with triangles
Decimation Operators

- Remove vertex
- Re-triangulate hole
  - Combinatorial DOFs
  - Sub-sampling
Decimation Operators

- Merge two adjacent triangles
- Define new vertex position
  - Continuous DOF
  - Filtering
Decimation Operators

- Collapse edge into one end point
  - Special vertex removal
  - Special edge collapse
- No DOFs
  - One operator per half-edge
  - Sub-sampling!
Edge Collapse
Edge Collapse
Edge Collapse
Edge Collapse
Edge Collapse
Edge Collapse
Edge Collapse
Edge Collapse
Edge Collapse
Edge Collapse
Priority Queue Updating
Incremental Decimation

- General Setup
- Decimation operators
- Error metrics
- Fairness criteria
- Topology changes
Local Error Metrics

• Local distance to mesh [Schroeder et al. 92]
  – Compute average plane
  – No comparison to original geometry
Global Error Metrics

• Simplification envelopes [Cohen et al. 96]
  – Compute (non-intersecting) offset surfaces
  – Simplification guarantees to stay within bounds
Global Error Metrics

• (Two-sided) Hausdorff distance: Maximum distance between two shapes
  – In general $d(A,B) \neq d(B,A)$
  – Computationally involved
Global Error Metrics

• Scan data: One-sided Hausdorff distance sufficient
  – From original vertices to current surface
Global Error Metrics

• Error quadrics [Garland, Heckbert 97]
  – Squared distance to planes at vertex
  – No bound on true error

\[ p_i^T Q_i p_i = 0, \ i=\{1,2\} \]

\[ Q_3 = Q_1 + Q_2 \]

solve \( v_3^T Q_3 v_3 = \min \]

\(< \varepsilon \ ? \ \rightarrow \ \text{ok} \)
Complexity

- $N =$ number of vertices
- Priority queue for half-edges
  - $6N \times \log(6N)$
- Error control
  - Local $O(1) \Rightarrow$ global $O(N)$
  - Local $O(N) \Rightarrow$ global $O(N^2)$
Incremental Decimation

- General Setup
- Decimation operators
- Error metrics
- Fairness criteria
- Topology changes
Fairness Criteria

• Rate quality of decimation operation
  – Approximation error
  – Triangle shape
  – Dihedral angles
  – Valence balance
  – Color differences
  – ...

Fairness Criteria

• Rate quality after decimation
  – Approximation error
  – Triangle shape
  – Dihedral angles
  – Valence balance
  – Color differences
  – ...

\[ \frac{r_1}{e_1} < \frac{r_2}{e_2} \]
Fairness Criteria

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[Diagram showing geometric shapes and network structures]
Fairness Criteria

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  – Approximation error
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  – Color differences
  – ...
Incremental Decimation

• General Setup
• Decimation operators
• Error metrics
• Fairness criteria
• Topology changes
Topology Changes?

- Merge vertices across non-edges
  - Changes mesh topology
  - Need *spatial neighborhood* information
  - Generates *non-manifold* meshes
Topology Changes?

- Merge vertices across non-edges
  - Changes mesh topology
  - Need *spatial neighborhood* information
  - Generates *non-manifold* meshes

![Manifold vs. Non-manifold](image)
Comparison

- **Vertex clustering**
  - fast, but difficult to control simplified mesh
  - topology changes, non-manifold meshes
  - global error bound, but often not close to optimum

- **Iterative decimation with quadric error metrics**
  - good trade-off between mesh quality and speed
  - explicit control over mesh topology
  - restricting normal deviation improves mesh quality
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Out-of-core Decimation

- Handle very large data sets that do not fit into main memory
- Key: Avoid random access to mesh data structure during simplification
- Examples
  - Garland, Shaffer: *A Multiphase Approach to Efficient Surface Simplification*, IEEE Visualization 2002
Multiphase Simplification

1. Phase: Out-of-core clustering
   - compute accumulated error quadrics and vertex representative for each cell of uniform voxel grid

2. Phase: In-core iterative simplification
   - compute fundamental quadrics
   - iteratively contract edge of smallest cost
Multiphase Simplification

1. Phase: Out-of-core clustering
   - compute accumulated error quadrics and vertex representative for each cell of uniform voxel grid

2. Phase: In-core iterative simplification
   - compute fundamental quadrics
   - use accumulated quadrics from clustering phase
   - iteratively contract edge of smallest cost

→ achieves a coupling between the two phases
Multiphase Simplification

Garland, Shaffer: *A Multiphase Approach to Efficient Surface Simplification*, IEEE Visualization 2002
Multiphase Simplification

Out-of-core Decimation

- Streaming approach based on edge collapse operations using QEM
- Pre-sorted input stream allows fixed-sized active working set independent of input and output model complexity

Out-of-core Decimation

• Randomized multiple choice optimization avoids global heap data structure

• Special treatment for boundaries required